Simulation in Alpine Skiing

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Data Collection

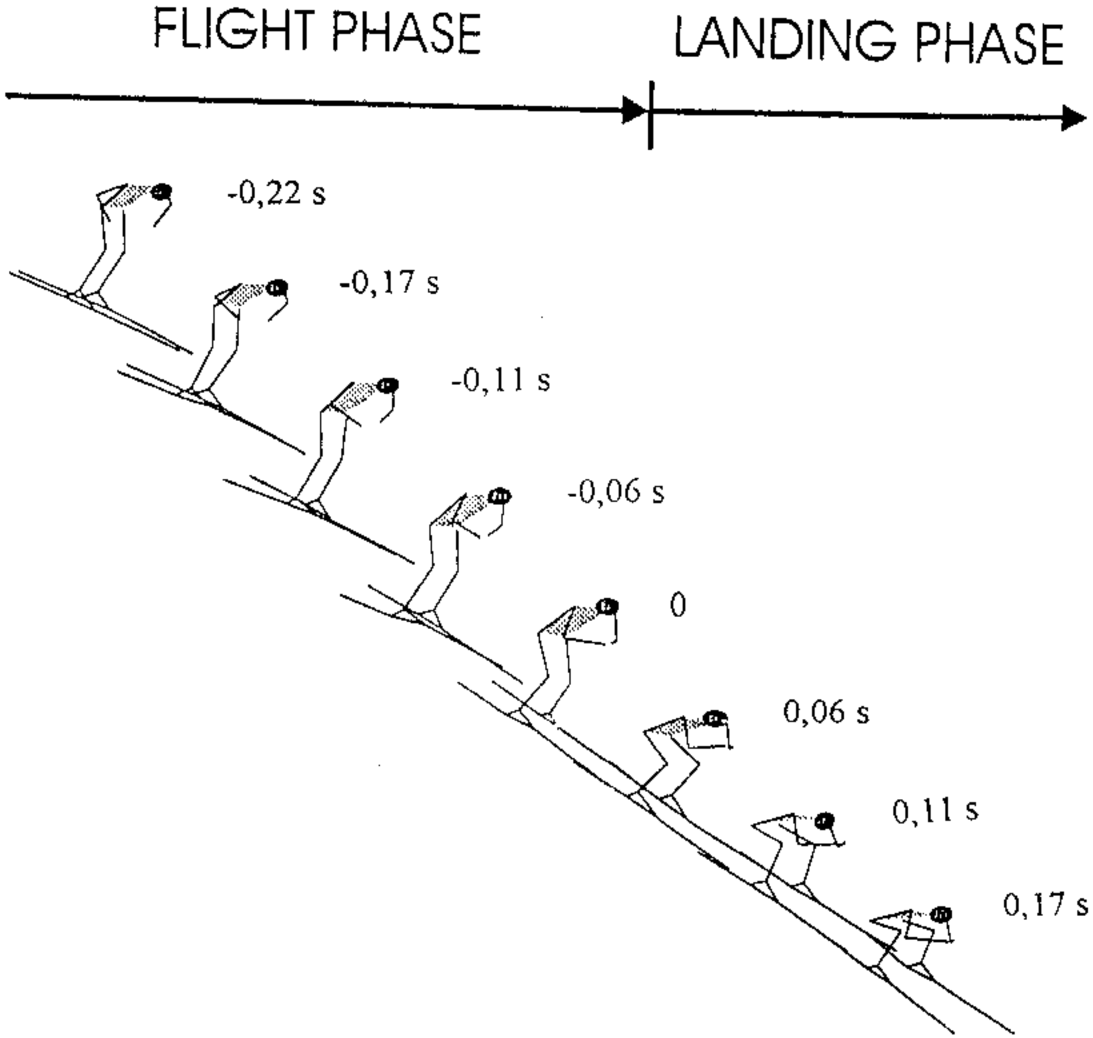
Trajectory of body points

Landing movement after jumps in
Alpine downhill skiing, Lillehammer

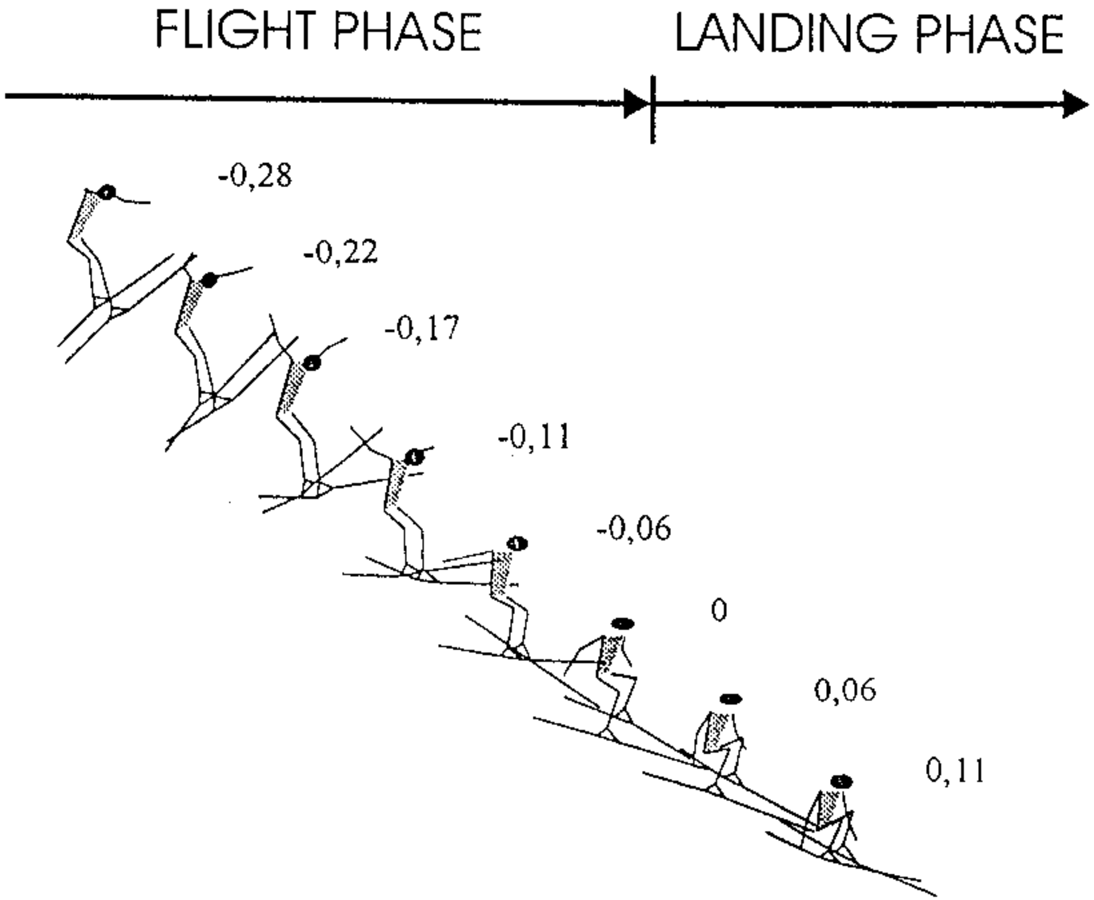
(Carved turns, Lech)

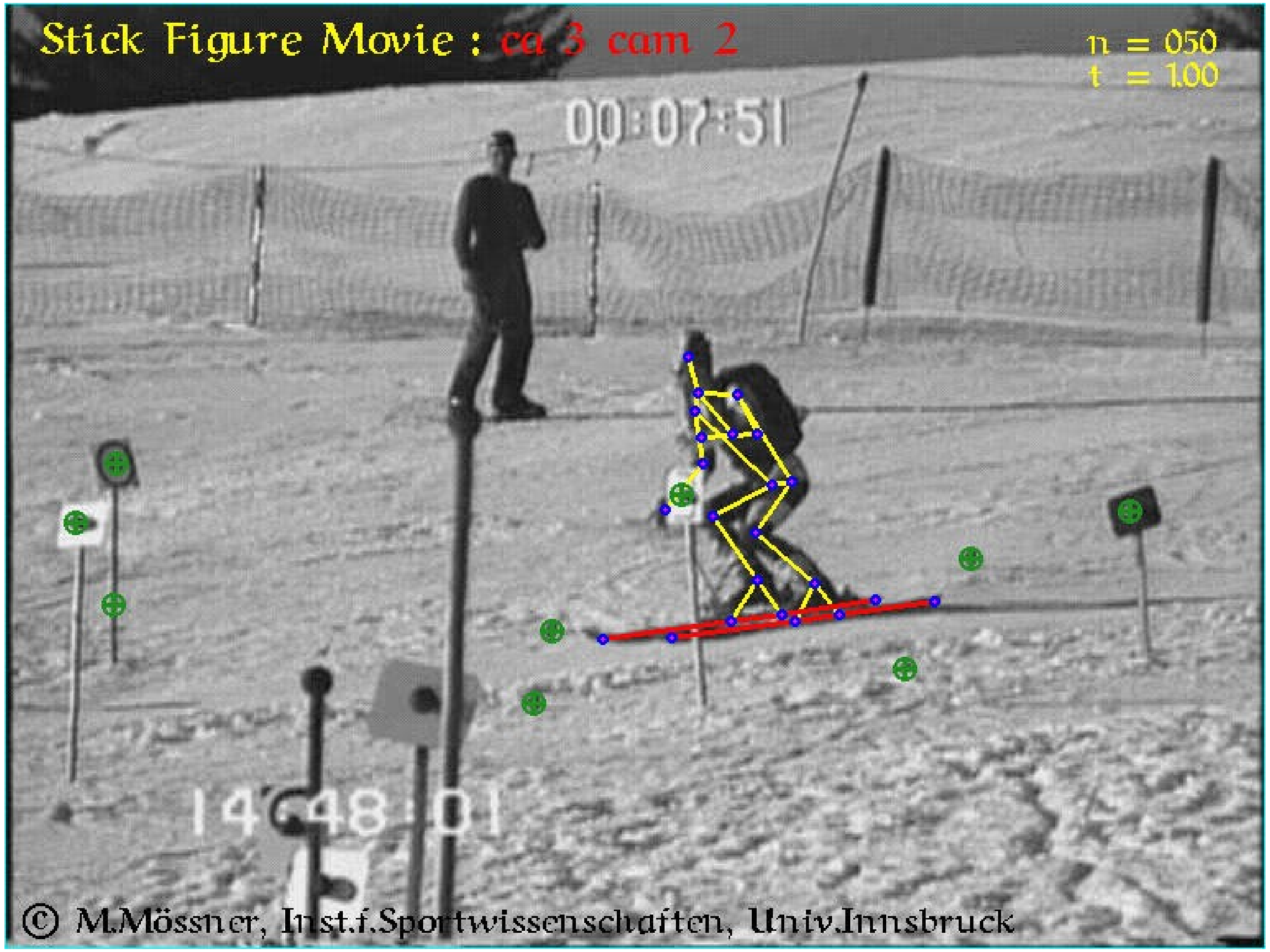
Turn, World Cup race, Streif, Kitzbühel

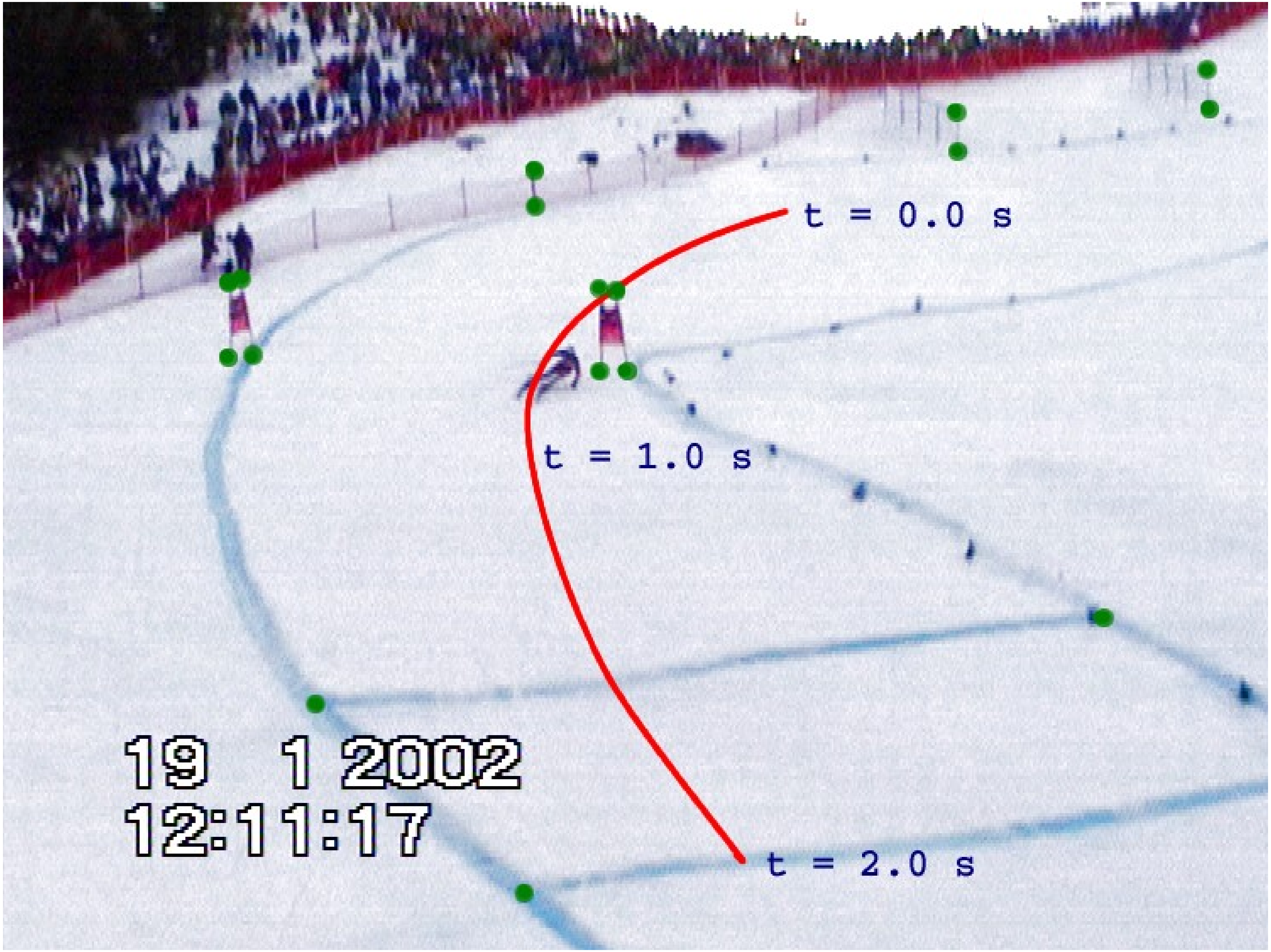
Optimal landing



Landing in backward position




Stick Figure Movie : ca 3 cam 2
n = 050
t = 1.00
00:07:51
14:48:01
© M.Mössner, Inst.f.Sportwissenschaften, Univ.Innsbruck


t = 0.0 s
t = 1.0 s
t = 2.0 s
19 1 2002
12:11:17

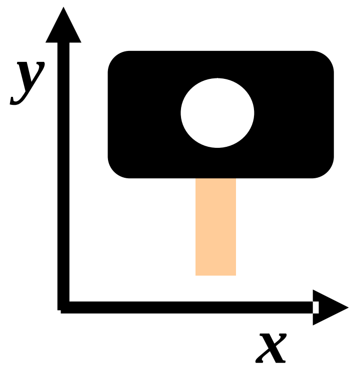
Direct linear transformation

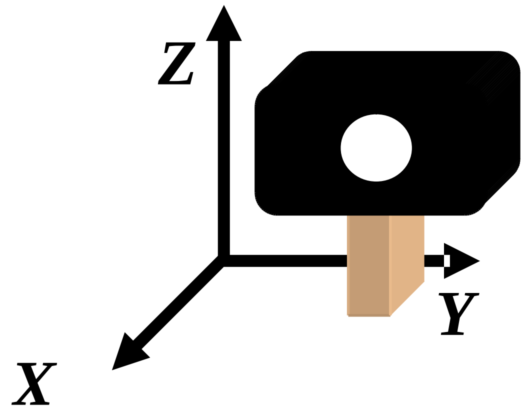
$$x = \frac{b_1 X + b_2 Y + b_3 Z + b_4}{b_9 X + b_{10} Y + b_{11} Z + 1} \quad y = \frac{b_5 X + b_6 Y + b_7 Z + b_8}{b_9 X + b_{10} Y + b_{11} Z + 1}$$

x,y image coordinates

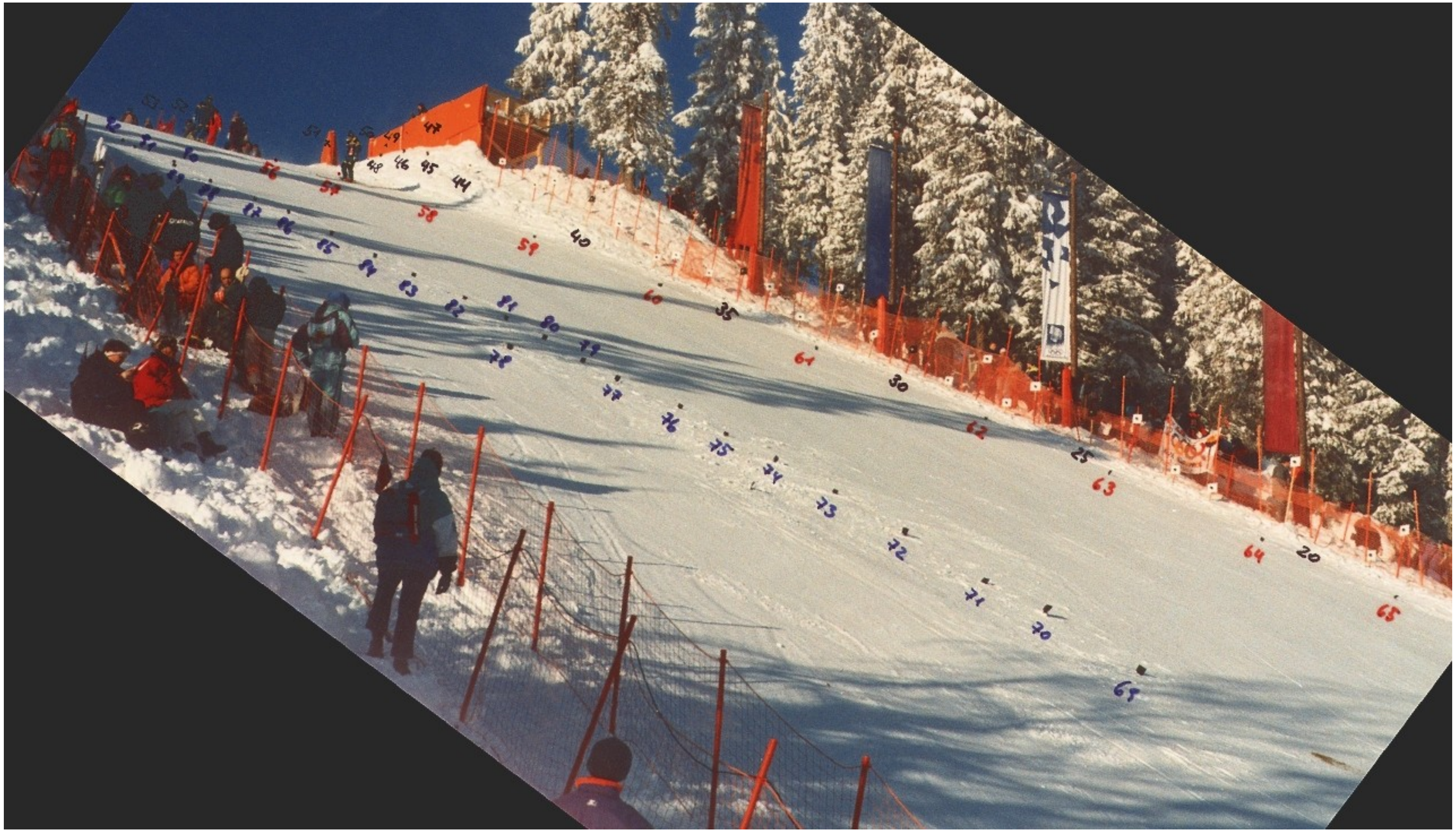
X,Y,Z object coordinates

b_i DLT-parameters





Control points at Russi jump



Camera position at Russi jump

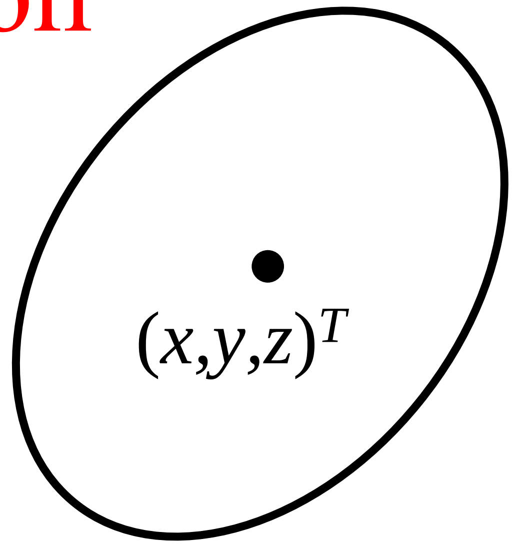
Video frame on PC

Unconstrained Newton-Euler equation of motion

$$m\ddot{x} = f_x$$

$$m\ddot{y} = f_y \qquad M\ddot{y} = f$$

$$m\ddot{z} = f_z$$



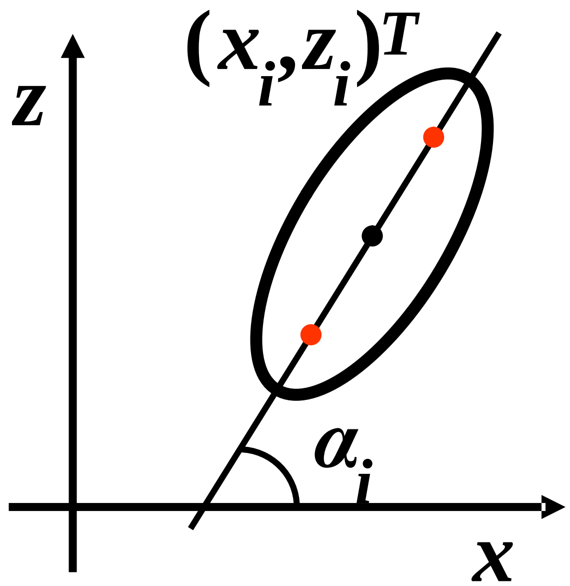
Rigid body

center of gravity: y=(x,y,z)T

$$J\dot{\omega} = n - \omega \times J\omega$$

Constrained equation of motion in 2D

$$m_i \ddot{x}_i = f_{xi} + r_{xi}$$
$$m_i \ddot{z}_i = f_{zi} + r_{zi}$$
$$I_i \ddot{\alpha}_i = f_{\alpha i} + r_{\alpha i}$$



unconstrained $r = 0$

Constrained Newton-Euler equation of motion

$$M\ddot{y} = f + r$$

f applied forces
r reaction forces

$$g(y,t) = 0$$

geometric constraint

$$G = g_y$$

$$r = -G^T\lambda$$

d‘Alembert‘s principle

$$M\ddot{y} + G^T\lambda = f$$

DAE

Constrained Newton-Euler equation of motion

$$M\ddot{y} + G^T \lambda = f$$ **DAE**

$$g(y,t) = 0$$ **index 3** **position level**

$$G\dot{y} = g^1$$ **index 2** **velocity level**

$$G\ddot{y} = g^2$$ **index 1** **acceler. level**

$$G = g_y$$

Equation of motion

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} T(u,t)\mathbf{v} \\ w \\ Mw + G^T\lambda - f \\ Gv - g^1 \end{bmatrix}$$

Index-2-DAE

Solved with RADAU 5 (Hairer-Wanner)
MATLAB-version of Ch. Engstler

Jumps in Alpine skiing

Ton van den Bogert

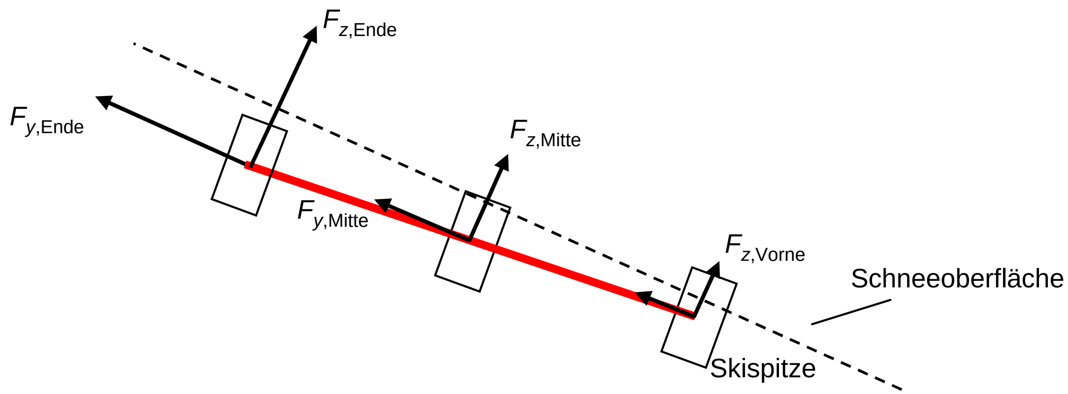
Karin Gerritsen

Kurt Schindelwig

Force between snow and ski

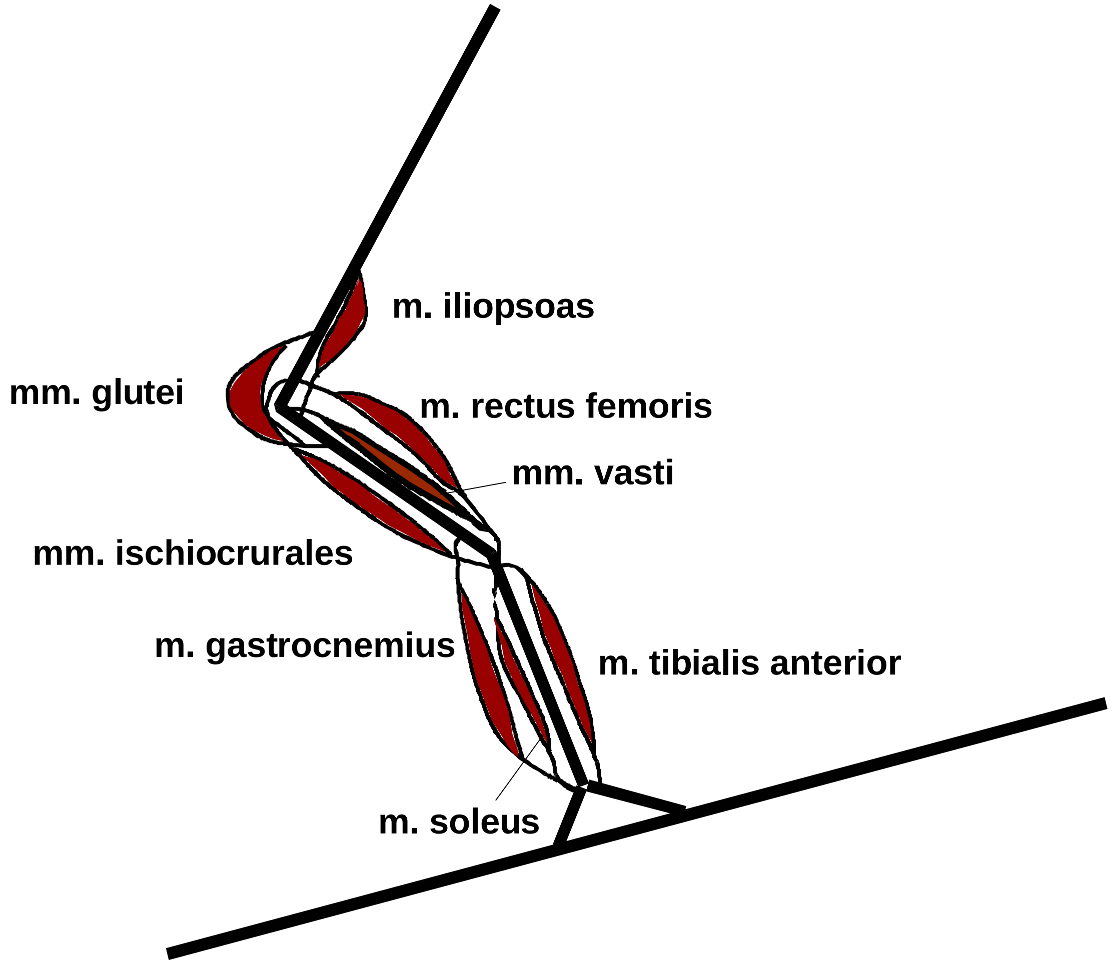
$$F_z = a|z|^p(1 - b\dot{z})$$

Force between snow and ski normal to snow surface



3 nonlinear viscoelastic contact elements

Musculo-skeletal model of a skier



muscle model van Soest, Bobbert 1993

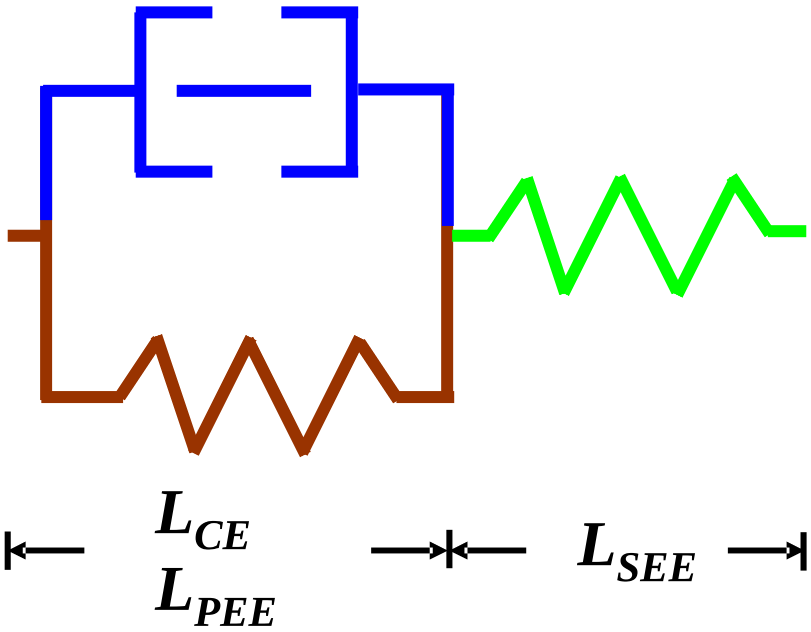
Muscle force

production of force – contractile element

ligaments - seriell elastic element

connective tissue - parallel elastic element

Muscle model of Hill



total length

$$\boldsymbol{L = L_{CE} + L_{SEE}}$$

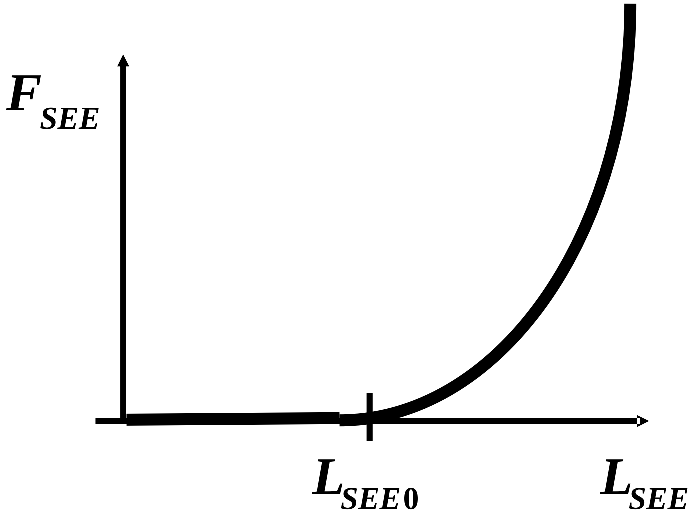
CE **contractile element**

SEE **seriell elastic element**

PEE **parallel elastic element**

Force of seriell-elastic elements

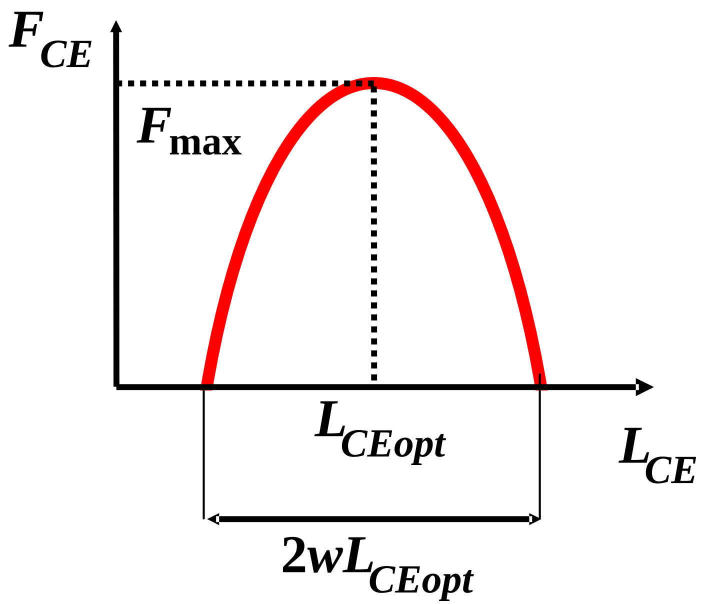
$$F_{SEE} = f(L_{SEE}) = f(L - L_{CE})$$



Force of parallel-elastic elements

$$F_{PEE} = f(L_{PEE}) = f(L_{CE})$$

Force-length-relation



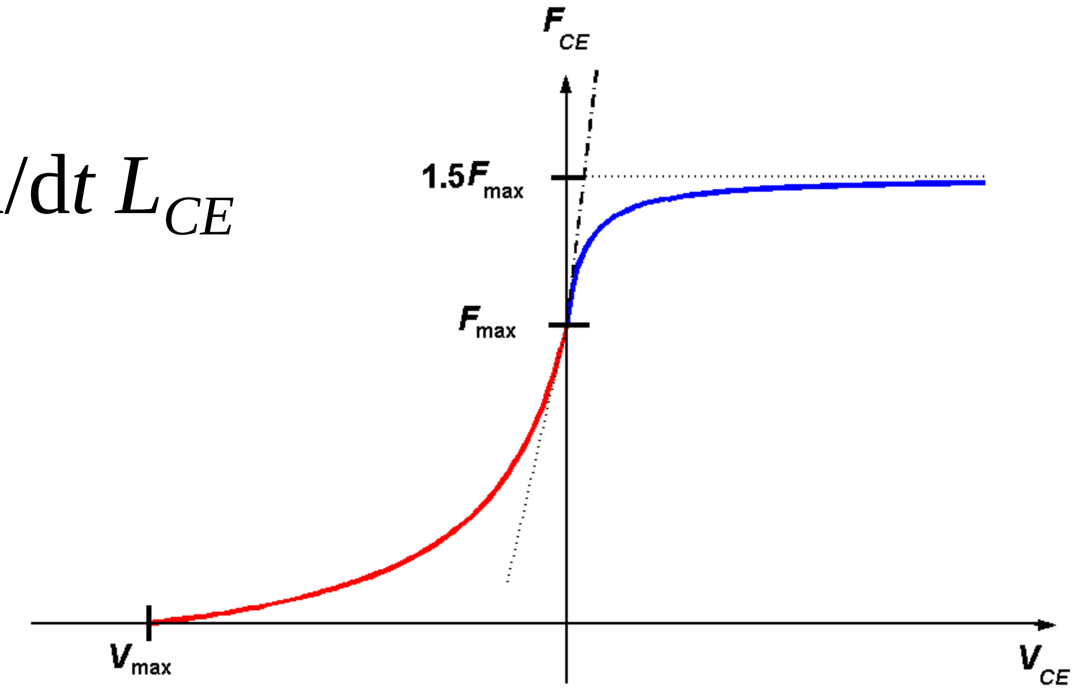
F_{max} maximal isometric force

isometric $v_{CE} = 0$

maximal activation $q = 1$

Force-velocity relation

$$v_{CE} = \mathrm{d/d}t \, L_{CE}$$



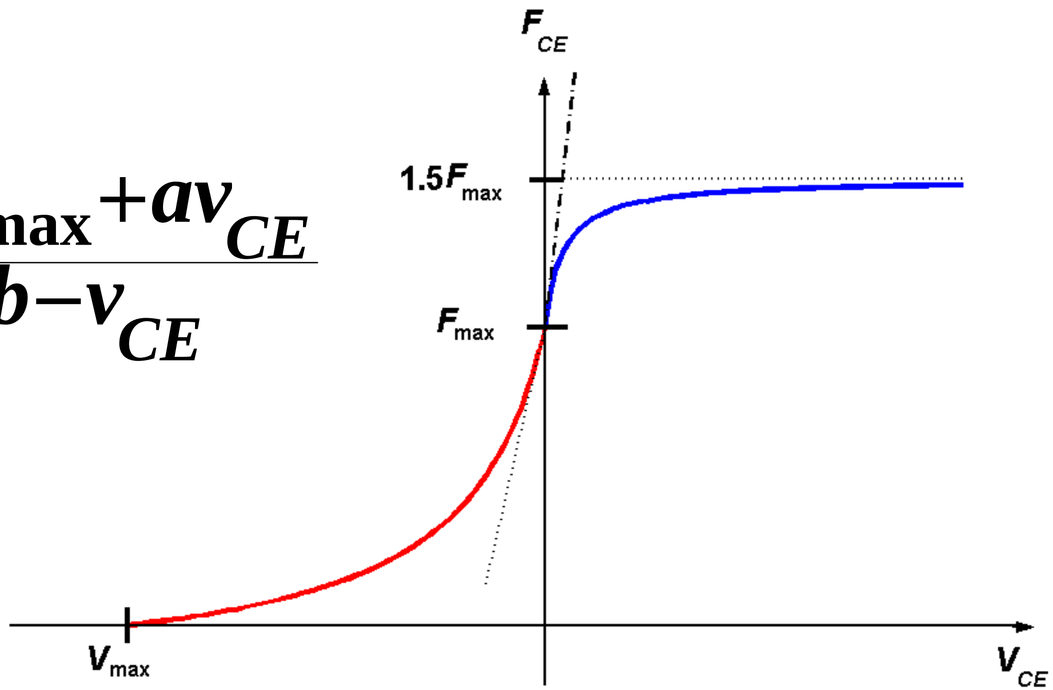
maximal activation q = 1

optimal muscle length $L_{CE} = L_{CEopt}$

Hill equation (1938)

Force-velocity relation

$$F_{CE} = \frac{bF_{\max} + av_{CE}}{b - v_{CE}}$$



concentric
contraction

Activation model (Hatze 1981) muscle activation

$$F_{CE} = qF_{\max} \qquad\qquad q = \frac{q_0 + [\rho(L_{CE})\gamma]^2}{1 + [\rho(L_{CE})\gamma]^2}$$

L_{CE} length of the contractile elements

γ calcium-ion concentration

$q_0 = 0.005$ value of the non activated muscle

$$\rho(L_{CE}) = 66200 \frac{1.90}{2.90^{\frac{L_{CEopt}}{L_{CE}}} - 1}$$

L_{CEopt} **optimal length of contractile elements**

Activation model (Hatze 1981)

Ordinary differential equation for
the calcium-ion concentration γ

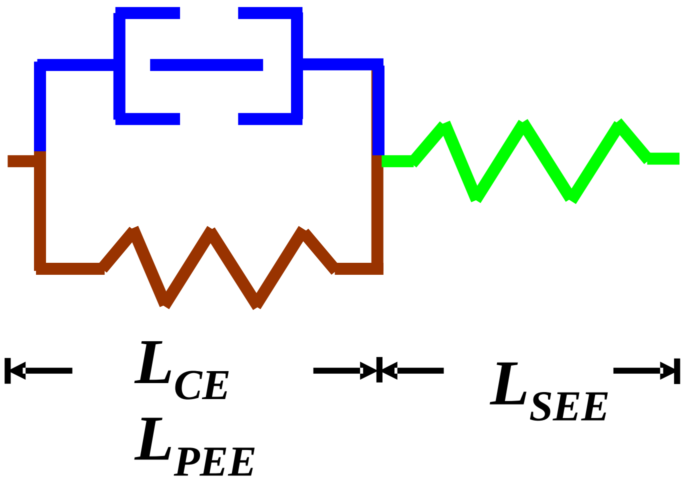
$$\frac{d\gamma}{dt} = m(c\eta - \gamma), \quad \gamma(0) = \gamma_0$$

Control parameter: relative stimulation rate

$$\eta = \frac{f}{f_{\max}} \qquad 0 \leq \eta \leq 1$$

f **stimulation rate,**
f_{max} **maximum stimulation rate**

Equilibrium



$$F_{CE} = F_{SEE} - F_{PEE} = f(L, L_{CE})$$

$$L_{SEE} = L - L_{CE}, \quad L_{PEE} = L_{CE}$$

$$F_{CE}(L, v_{CE}, q) = f(L, L_{CE})$$

Solving for v_{CE}

$$v_{CE} = \mathrm{d/d}t\, L_{CE} = f_H(L, L_{CE}, q(\gamma, L_{CE}))$$

State of a muscle

three state variables

$$L, L_{CE}, \gamma$$

L actual muscle length

L_{CE} length of the contractile element

γ calcium-ion conzentration

Force of muscle-ligament complex

according to Hill-Modell

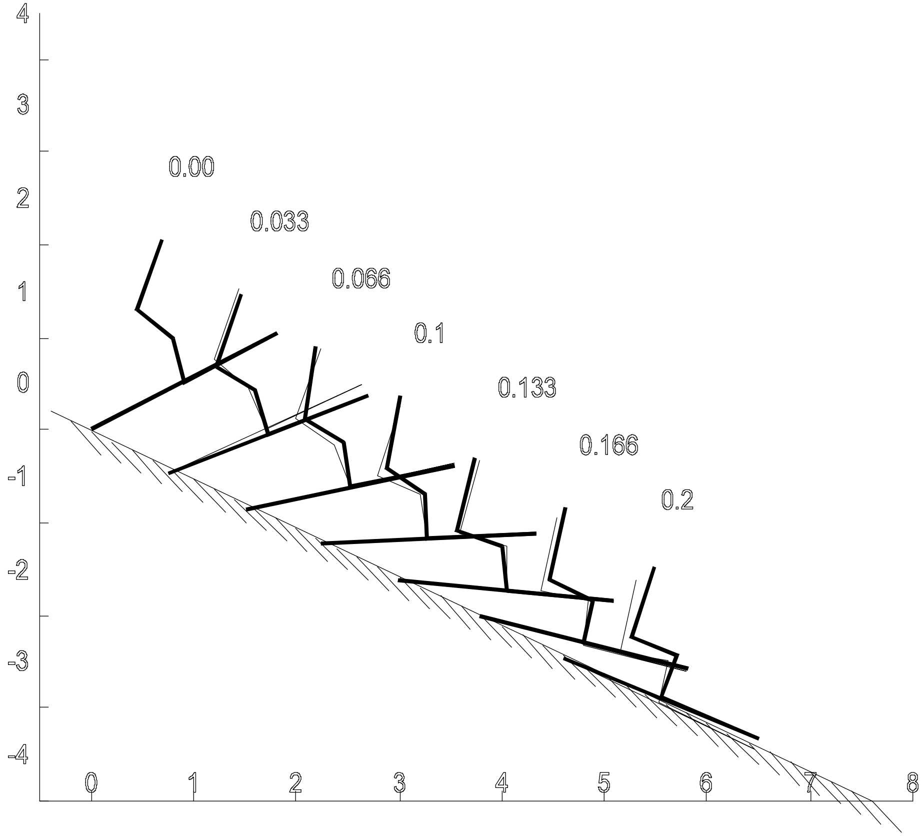
Input: L, L_{CE}, γ

compute equivalent torque

muscle force times
lever arm D_k for joint k

D_k constant

Comparison measured (——) and simulated (——) landing movement



Turns in Alpine skiing

Simulation with DADS

Peter Lugner
Franz Bruck
Techn. University, Vienna

Trajectory of a ski racer

$$x(t)=(X(t), Y(t), Z(t))^T$$

position as a function of time

Mean value between the toe pieces of the left and right binding

Track

$$Z-h(X,Y)=0$$
$$Y-s(X)=0$$

Position constraint

$$g(x,t)=0$$

Equation of Motion

Skier modelled as a mass point

descriptor form dependent coordinates x

Differential-Algebraic Equation DAE

$$m\ddot{x} = f + r \qquad \text{ODE}$$

$$g(x,t) = 0 \qquad \text{algebraic equation}$$

f applied forces

r reaction forces $r = -g_x^T \lambda$

Applied forces

gravity snow friction drag

$$f = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} - \mu N t - \frac{1}{2} c_d A \rho v^2 t$$

t unit vector in tangential direction $t = \frac{v}{\| v \|}$

μ friction coefficient

N normal force $N = \|r\|$

$c_d A$ drag area

ρ density

v velocity

Snow friction and drag area

piecewice constant values

$$t_{i-1} \leq t \leq t_i \qquad \mu_i \qquad (c_d A)_i$$

determination of μ_i , $(c_d A)_i$, t_i

by a least squares argument

$$\sum \| x(t_i) - x_i \|^2 \qquad \text{minimum}$$

$x(t_i)$ DAE-solution at time t_i

x_i smoothed DLT-result at time t_i

Software for Computation

Computations were performed in MATLAB

DAE-solver

RADAU5 of Hairer-Wanner

MATLAB-Version by Ch. Engstler

Optimization problem

Nelder-Mead simplex algorithmus

Results

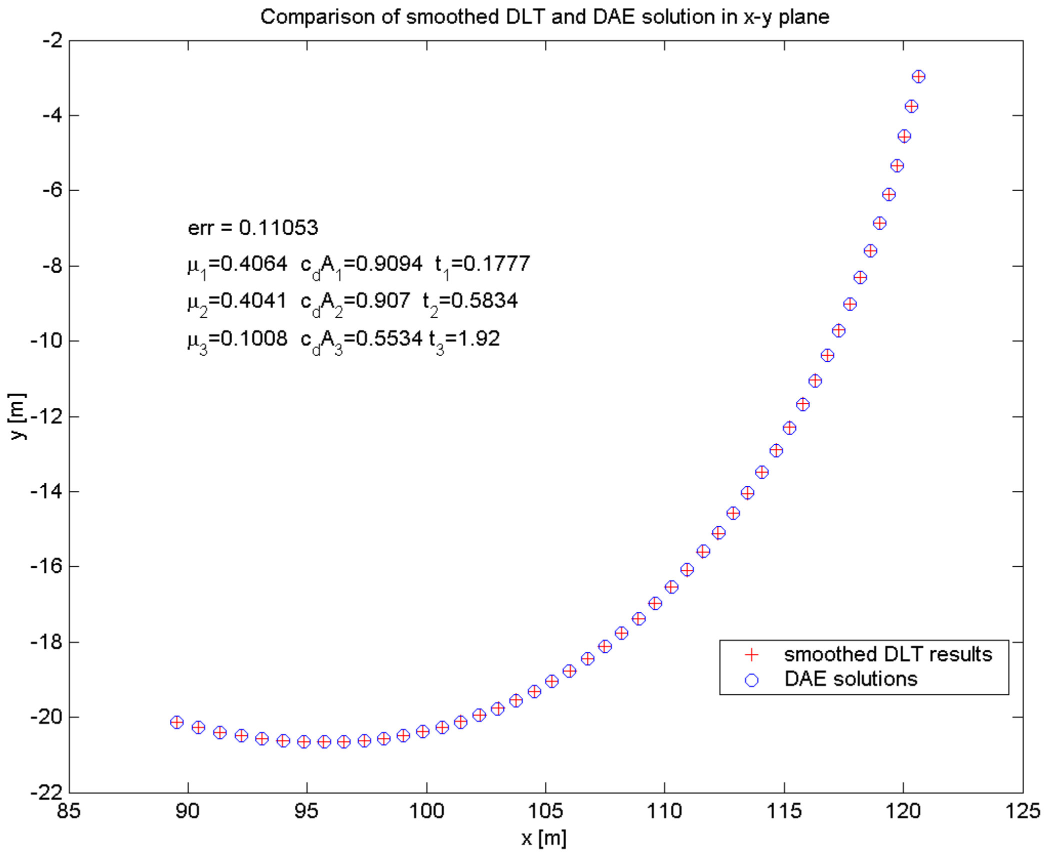
truncated values

	$t<0.58$	t>0.58
μ	0.40	0.10
$c_d A$	0.90	0.55

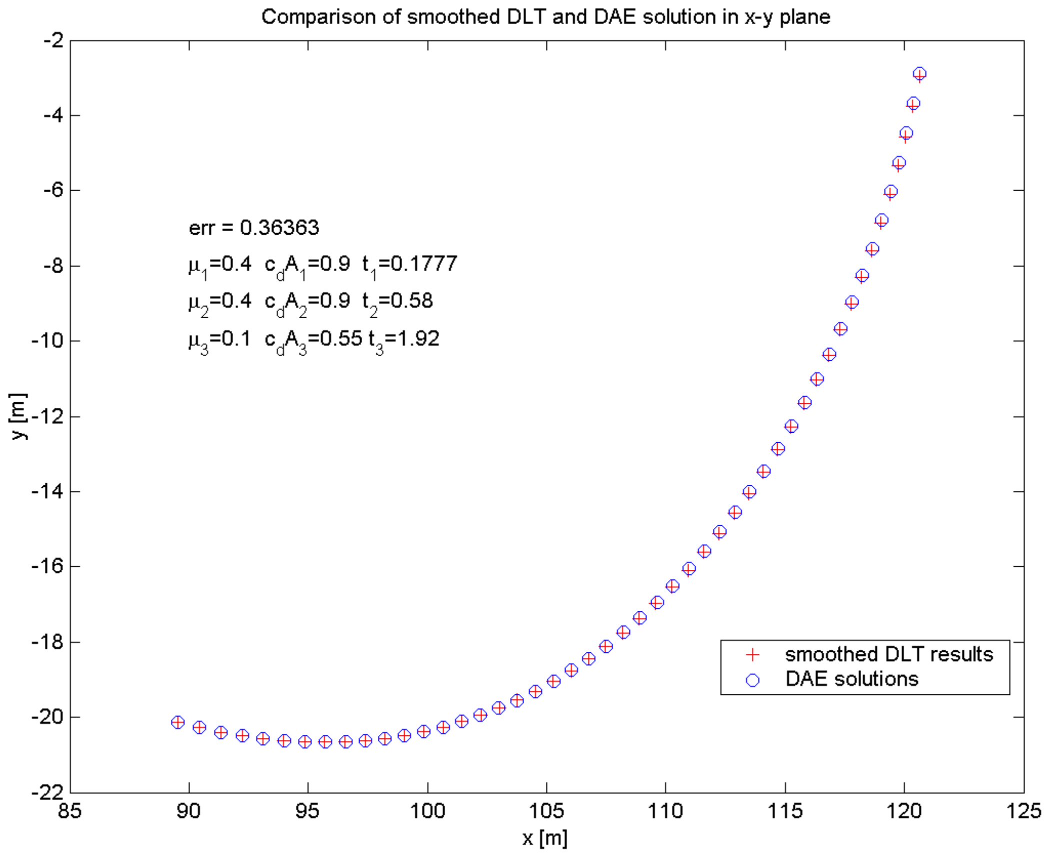
more exact values

[0 , 0.1777]	$\mu_1=0.4064$	$(c_d A)_1=0.9094$
[0.1777, 0.5834]	$\mu_2=0.4041$	$(c_d A)_2=0.9070$
[0.5834, 1.9200]	$\mu_1=0.1008$	$(c_d A)_1=0.5534$

Comparison more exact values



Comparison truncated values



Conclusions

In Alpine skiing biomechanical studies under race conditions are possible. The results are reasonable, although circumstances for data collection are not optimal: no markers, position of control points must not disturb the racers, difficulties with commercial rights

Results like loading of the anterior cruciate ligament (ACL) as function of velocity or inclination of the slope during landing or the possibility of a rupture of the ACL without falling are interesting applications in medicine.

Informations on snow friction and drag in race conditions are interesting results, but a video analysis is expensive (digitizing the data, geodetic surveying).

Applications

Determination of an optimal trajectory

Virtual skiing, with vibration devices, in analogy to flight simulators